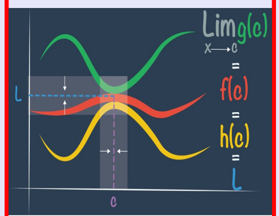
Calculus I

Lecture 43



Feb 19-8:47 AM

Find a point on the graph of $xy=8$ that is closest to $(3,0)$

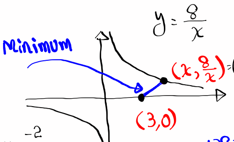
Hyperbola

$y=\frac{8}{x}$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(x-3)^2+\left(\frac{8}{x}-0\right)^2}$

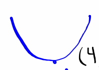
$=\sqrt{(x-3)^2+\frac{64}{x^2}}$



Let $f(x)=(x-3)^2+\frac{64}{x^2}$ $\quad 64x^{-2}$

$f'(x)=2(x-3)^1\cdot 1-\frac{128}{x^3}$ $\quad 128x^{-3}$

$f''(x)=2+128\cdot 3x^{-4}=2+\frac{384}{x^4}>0 \to$ C.U.



$f'(x)=0$ $\quad 2(x-3)-\frac{128}{x^3}=0$

$2x^3(x-3)-128=0$

$x^4-3x^3-64=0$

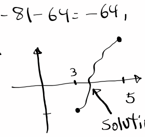
Polynomial, Cont. & Diff. $(-\infty,\infty)$

Pre Calc & Calc.

$f(1)=1-3-64=-66$, $f'(2)=16-24-64=-72$

$f'(3)=81-81-64=-64$, $f'(5)=625-375-64=+$

By I.V.T.



$f'(4)=4^4-3(4)^3-64$

$=256-192-64=0$

4 is the Solution

$f'(4)=0 \to$ C.P. at $x=4$

Apr 30-9:40 AM

Solve $x^4-3x^3-64=0$ by Newton's Method.

Newton's eqn $\quad x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$

$f(x)=x^4-3x^3-64$

$f'(x)=4x^3-9x^2$

$$x_{n+1}=x_n-\frac{x_n^4-3x_n^3-64}{4x_n^3-9x_n^2}$$

$$=\frac{x_n(4x_n^3-9x_n^2)-(x_n^4-3x_n^3-64)}{4x_n^3-9x_n^2}$$

$$x_{n+1}=\frac{3x_n^4-6x_n^3+64}{4x_n^3-9x_n^2}$$

$x_1=5$

$$x_2=\frac{3(5)^4-6(5)^3+64}{4(5)^3-9(5)^2}=4.32$$

$$x_3=\frac{3(4.32)^4-6(4.32)^3+64}{4(4.32)^3-9(4.32)^2}=4.05$$

$$x_4=\frac{3(4.05)^4-6(4.05)^3+64}{4(4.05)^3-9(4.05)^2}=4.00$$

Solution is $x=4$

$$x_5=\frac{3(4)^4-6(4)^3+64}{4(4)^3-9(4)^2}=4$$

May 1-8:54 AM

Use Calc method to graph $f(x)=\dfrac{1}{x^2-9}$

Domain $\to$ $x\neq\pm 3$, $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

V.A. $x=\pm 3$

H.A. $\lim_{x\to\pm\infty} f(x)=0 \to y=0$

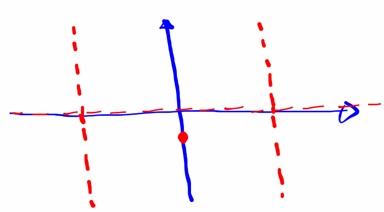
even function

$f(-x)=\dfrac{1}{(-x)^2-9}=\dfrac{1}{x^2-9}=f(x)$ Symmetric w/t y-axis

y-Int. $\to x=0 \to f(0)=\frac{-1}{9}$ $\quad (0,-\frac{1}{9})$

x-Int $\to y=0 \to f(x)=0 \to$ No Soln

NO x-Ints



May 1-9:06 AM

$f(x) = \frac{1}{x^2-9}$ $\qquad$ $f(x) = (x^2-9)^{-1}$

$f'(x) = -1(x^2-9)^{-2} \cdot 2x$ $\qquad$ $f'(x) = \frac{-2x}{(x^2-9)^2}$

$f'(x) = -2x(x^2-9)^{-2}$

$f'(x) = 0 \rightarrow x = 0$

$f'(x)$ und. $\rightarrow x = \pm 3$

$$f''(x) = -2\left[1(x^2-9)^{-2} + x \cdot -2(x^2-9)^{-3} \cdot 2x\right]$$

$$= -2\left[(x^2-9)^{-2} - 4x^2(x^2-9)^{-3}\right]$$

$$= -2(x^2-9)^{-3}\left[(x^2-9)^{1} - 4x^2\right]$$

$$= \frac{-2}{(x^2-9)^3} \cdot (-3x^2 - 9) = \frac{-2 \cdot -3(x^2+3)}{(x^2-9)^3}$$

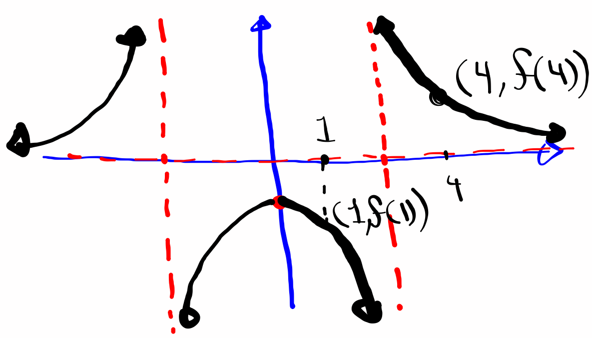
$$f''(x) = \frac{6(x^2+3)}{(x^2-9)^3}$$

$f''(x) \neq 0$

$f''(x)$ und. at $x = \pm 3$

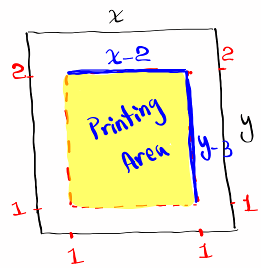
May 1-9:11 AM

x	$-\infty$		-3		0		3		∞
$f'(x)$		+	o	+	•	−	o	−	
$f''(x)$		+	o	−		−	o	+	
$f(x)$									



May 1-9:18 AM

A poster has an area of 180 in^2.
1 in. margin at bottom and Sides, and 2 in. margin at the top. It is rectangular.
What dimensions has the largest printing area?



$x \cdot y = 180 \rightarrow y = \frac{180}{x}$

Printing Area is
$(x-2)(y-3)$
Maximize it
$xy - 3x - 2y + 6$
$= 180 - 3x - 2y + 6$
$= 186 - 3x - 2\left(\frac{180}{x}\right)$

$f(x) = 186 - 3x - \frac{360}{x}$

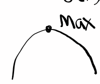
$f'(x) = -3 + \frac{360}{x^2}$ $\qquad f''(x) = \frac{-720}{x^3}$

$-3 + \frac{360}{x^2} = 0$ $\qquad$ Since $x>0$, $f''<0$

$f(x)$ is C.D.

$-3x^2 + 360 = 0$
$x^2 - 120 = 0$

$\rightarrow x = \sqrt{120}$
Now find y
$y = \frac{180}{\sqrt{120}}$

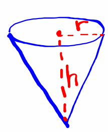


$f'(x) = 0$

May 1-9:26 AM

A Cone-shaped paper drinking cup is to be made to hold $\boxed{27\ cm^3}$ of water.

Find height and radius of the cup with Smallest amount of paper.



$V = \frac{1}{3}\pi r^2 h$

$\boxed{A = \pi r\sqrt{r^2 + h^2}}$
Minimized

$27 = \frac{1}{3}\pi r^2 h$
$81 = \pi r^2 h$
$h = \frac{81}{\pi r^2}$

$f(r) = \pi r \cdot \sqrt{r^2 + \frac{81^2}{\pi^2 r^4}}$

$= \pi r \cdot \sqrt{\frac{\pi^2 r^6 + 81^2}{\pi^2 r^4}} = \sqrt{\cancel{\pi^2}\cancel{r^2} \cdot \frac{\pi^2 r^6 + 81^2}{\cancel{\pi^2}\cancel{r^4}_{r^2}}}$

$f(r) = \sqrt{\frac{\pi^2 r^6 + 81^2}{r^2}}$

To minimize $f(r)$, we must minimize the radicand

$g(r) = \frac{\pi^2 r^6 + 81^2}{r^2}$ $\qquad g(r) = \pi^2 r^4 + 81 r^{-2}$

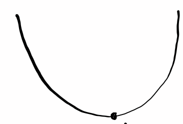
May 1-9:37 AM

$g(r) = \pi^2 r^4 + 81 r^{-2}$

$g'(r) = \pi^2 \cdot 4r^3 - 162 r^{-3}$

$g''(r) = \pi^2 \cdot 12r^2 + 486 r^{-4} = \pi^2 \cdot 12 r^2 + \frac{486}{r^4} > 0$

$g(r)$ is C.U.



$g'(r) = 0$

$4\pi^2 r^3 - \frac{162}{r^3} = 0$

$4\pi^2 r^6 - 162 = 0$

$2\pi^2 r^6 - 81 = 0$

$r^6 = \frac{81}{2\pi^2} \qquad r = \sqrt[6]{\frac{81}{2\pi^2}}$ ≈ Radius

Recall $h = \frac{81}{\pi r^2} = \frac{81}{\pi} \cdot \frac{1}{\sqrt[3]{\frac{81}{2\pi^2}}}$ ≈ height

May 1-9:47 AM